# AGNT Essentials - problems set 1

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**Problem 1** *Let $F{:}\mathcal{A} \longrightarrow \mathcal{B}$ be a functor.*

*(a) Suppose that $F$ is an equivalence. Prove that $F$ is fully faithful and essentially surjective on objects.*

*(b) Now suppose instead that $F$ is fully faithful and essentially surjective on objects. For each $B \in \mathcal{B}$ choose an object $G(B)$ of $\mathcal{A}$ and an isomorphism $\epsilon_B : F(G(B)) \longrightarrow B$. Prove that $G$ extends to a functor in such a way that $(\epsilon_B)_(B \in \mathcal{B})$ is a natural isomorphism $FG \Longrightarrow 1_{\mathcal{B}}$. Then construct a natural isomorphism $1_{\mathcal{A}} \to GF$, thus proving that $F$ is an equivalence.*

**Problem 2** *Show that equivalence of categories is an equivalence relation.*

**Problem 3**

*(a) Give an example of a category **not** isomorphic to its opposite.*

*(b) Give an example of a category isomorphic to it's opposite*

**Problem 4** *Give three examples of categories so that none of them is equivalent to any other.*

**Problem 5** *Why is it that the analog construction of Russell's Paradox does not lead to a contradiction in Category Theory?*